Algebra Basics

Before you can really start algebra, there are some basics you need to learn.

Letters Multiplied Together

Watch out for these combinations of letters in algebra that regularly catch people out:

1) abc means $a \times b \times c$. The ×'s are often left out to make it clearer.
2) gn^2 means $g \times n \times n$. Note that only the n is squared, not the g as well — e.g. πr^2 means $\pi \times r \times r$.
3) $(gn)^2$ means $g \times g \times n \times n$. The brackets mean that BOTH letters are squared.
4) $p(q-r)^3$ means $p \times (q-r) \times (q-r) \times (q-r)$. Only the brackets get cubed.
5) Avoid writing things like -3^2. It should either be $(-3)^2 = 9$, or $-(3^2) = -9$ (you'd usually take -3^2 to be -9).

Terms

Before you can do anything else with algebra, you must understand what a term is:

KEY TERM

A term is a collection of numbers and letters, all multiplied/divided together.

Terms are separated by + and – signs. Every term has a + or – attached to the front of it.

If there's no sign in front of the first term, it means there's an invisible + sign.

$$4xy + 5x^2 - 2y + 6y^2 + 4$$

'xy' term   'x^2' term   'y' term   'y^2' term   'number' term

Simplifying or 'Collecting Like Terms'

To simplify an algebraic expression, you combine 'like terms' — terms that have the same combination of letters (e.g. all the x terms, all the y terms, all the number terms etc.).

EXAMPLE:

Simplify $2x - 4 + 5x + 6$

number terms

Invisible + sign

$$2x \; -4 \; +5x \; +6 = +2x \; +5x \; -4 \; +6$$

x-terms

$$= 7x \quad +2 \quad = 7x + 2$$

1) Put circles round each term — be sure you include the +/– sign in front of each.
2) Then you can move the circles into the best order so that like terms are together.
3) Combine like terms.

These are the basics of everything in algebra

REVISION TIP

Algebra isn't easy, so if you don't get these basics learned now, you'll be really confused later on. Always remember — every term has a + or – stuck to the front (even if it's invisible).

Powers

You've already seen powers on page 6 — have a look back at that page if you need a reminder. There are some special rules for powers that you need to learn — starting with the eight on this page.

Learn these **Eight** Rules

1) When MULTIPLYING, you ADD THE POWERS.

e.g. $3^4 \times 3^6 = 3^{4+6} = 3^{10}$, $a^2 \times a^7 = a^{2+7} = a^9$

Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

2) When DIVIDING, you SUBTRACT THE POWERS.

e.g. $5^4 \div 5^2 = 5^{4-2} = 5^2$, $b^8 \div b^5 = b^{8-5} = b^3$

Simple algebraic fractions can be simplified using these power rules.

E.g. $\frac{p^3q^6}{p^2q^3} = p^3q^6 \div p^2q^3$
$= p^{3-2}q^{6-3}$
$= pq^3$

3) When RAISING one power to another, you MULTIPLY THEM.

e.g. $(3^2)^4 = 3^{2\times4} = 3^8$, $(c^3)^6 = c^{3\times6} = c^{18}$

4) $x^1 = x$, ANYTHING to the POWER 1 is just ITSELF.

e.g. $3^1 = 3$, $d \times d^3 = d^1 \times d^3 = d^{1+3} = d^4$

5) $x^0 = 1$, ANYTHING to the POWER 0 is just 1.

e.g. $5^0 = 1$, $67^0 = 1$, $e^0 = 1$

6) $1^x = 1$, 1 TO ANY POWER is STILL JUST 1.

e.g. $1^{23} = 1$, $1^{89} = 1$, $1^2 = 1$

Powers can also be called 'indices' (or 'index' if it's just one).

7) FRACTIONS — Apply the power to both TOP and BOTTOM.

e.g. $\left(1\frac{3}{5}\right)^3 = \left(\frac{8}{5}\right)^3 = \frac{8^3}{5^3} = \frac{512}{125}$, $\left(\frac{u}{v}\right)^5 = \frac{u^5}{v^5}$

8) NEGATIVE Powers — Turn it Upside Down.

People find it difficult to remember this one — whenever you see a negative power you need to think: "That means turn it the other way up and make the power positive".

e.g. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $a^{-4} = \frac{1}{a^4}$, $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{+2} = \frac{5^2}{3^2} = \frac{25}{9}$

REVISION TASK

Rules 1 & 2 only work for powers of the same number

If you can add, subtract and multiply, there's nothing on this page you can't do — as long as you learn the rules. Try copying them over and over until you can do it with your eyes closed.

Powers

These **Two** Rules are a bit more **Tricky**

9) FRACTIONAL POWERS

See page 6 for more on roots. You'll be able to use your calculator in the exam if you get a root that's really hard to evaluate.

The power $\frac{1}{2}$ means Square Root. e.g. $25^{\frac{1}{2}} = \sqrt{25} = 5$

The power $\frac{1}{3}$ means Cube Root. $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

The power $\frac{1}{4}$ means Fourth Root etc. $81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

The one to really watch is when you get a negative fraction like $49^{-\frac{1}{2}}$ — people get mixed up and think that the minus is the square root, and forget to turn it upside down as well.

e.g. $49^{-\frac{1}{2}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$

10) TWO-STAGE FRACTIONAL POWERS

With fractional powers like $64^{\frac{5}{6}}$, always split the fraction into a root and a power, and do them in that order: root first, then power: $(64)^{\frac{1}{6}\times 5} = (64^{\frac{1}{6}})^5 = (2)^5 = 32$.

You can use **Fractional Powers** with **Algebra** too

EXAMPLE: **Simplify $2xy^{\frac{3}{2}} \times 3x^{-\frac{5}{6}}y^{-\frac{3}{2}}$**

Just deal with each bit separately:

$$2xy^{\frac{3}{2}} \times 3x^{-\frac{5}{6}}y^{-\frac{3}{2}} = (2 \times 3)(x \times x^{-\frac{5}{6}})(y^{\frac{3}{2}} \times y^{-\frac{3}{2}})$$

$$= (2 \times 3)x^{1+(-\frac{5}{6})}y^{\frac{3}{2}+(-\frac{3}{2})} = 6x^{1-\frac{5}{6}}y^{\frac{3}{2}-\frac{3}{2}} = 6x^{\frac{1}{6}}$$

$y^{\frac{3}{2}-\frac{3}{2}} = y^0 = 1$

EXAMPLE: **Evaluate $\left(\frac{x^6}{27}\right)^{\frac{2}{3}}$**

1) Break down the two-stage fractional power into a root and a power. Remember to do everything to both the top and the bottom.

$$\left(\frac{x^6}{27}\right)^{\frac{2}{3}} = \left(\left(\frac{x^6}{27}\right)^{\frac{1}{3}}\right)^2 = \left(\frac{(x^6)^{\frac{1}{3}}}{27^{\frac{1}{3}}}\right)^2 = \left(\frac{x^{(6\times\frac{1}{3})}}{\sqrt[3]{27}}\right)^2$$

This uses power rule 3 — when you raise one power to another, you multiply them together.

2) Work out the root and deal with the powers that are left.

$$= \left(\frac{x^2}{3}\right)^2 = \frac{(x^2)^2}{3^2} = \frac{x^4}{9}$$

These power rules might be a bit trickier — but they are essential

People often get fractional powers wrong, so you should write down these rules and make sure you learn them. Then, if they come up in the exam, you won't have anything to worry about.